Fault-Tolerant Supervisory Control in Terms of Formal Languages

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Discrete-Event Systems

Supervisory Control

Naive Fault-Tolerant Control

Active Fault-Tolerant Control

Post-Fault Recovery

Fault-Hiding Approach

Formal Languages

- Σ^* set of all finite strings over Σ
- empty string $\epsilon \in \Sigma^*$, $\epsilon \notin \Sigma$
- $*$-language $M \subseteq \Sigma^*$
- prefix operator $\operatorname{pre} s := \{\, t \mid \exists\, r : tr = s \,\}$
- note that $\operatorname{pre} M = \{\, t \mid \exists\, r : tr \in M \,\}$
- M is closed $:\Leftrightarrow \;\; M = \operatorname{pre} M$

A natural domain for the interpretation of liveness properties are ω-languages, i.e., sets of infinite-length strings $w \in \Sigma^\omega$.

If there are no deadlocks, we may use

$$\mathcal{M} := \{w \in \Sigma^\omega \mid \; \operatorname{pre} w \subseteq \operatorname{pre} M \;\}$$

to model the process w.r.t. infinite time.

If, in addition, there are no livelocks, we may consider

$$\mathcal{L} := \{w \in \Sigma^\omega \mid \; \|(\operatorname{pre} w) \cap L\| = \infty\}.$$

to model the process w.r.t. infinite time.

Control patterns

$$\Gamma := \{\, \gamma \subseteq \Sigma \mid \Sigma_{uc} \subseteq \gamma \,\}$$

Projection

- natural projection $p_o : \Sigma^* \to \Sigma_o^*$
  read "removes all symbols not from Σ_o"
- for languages take pint-wise images
- set-valued inverse $p_o^{-1} : \Sigma_o^* \rightsquigarrow \Sigma^*$
  read "arbitrarilly inserts symbols from Σ_{uo}"

Language Quotient

$$K/E := \{\, s \mid \exists\, t \in E \;:\; st \in K \,\}\,.$$

Language Convergence

K *finitely converges* to E if there exists a uniform bound k such that every $s \in K$ can be decomposed

$$s = vw,\; w \in E, \text{ and } |v| \leq k.$$

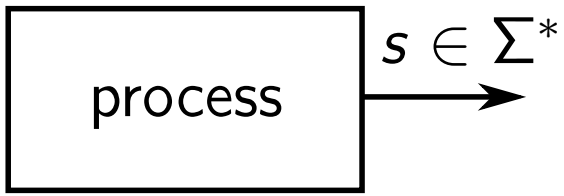
This is written $E \Leftarrow K$.

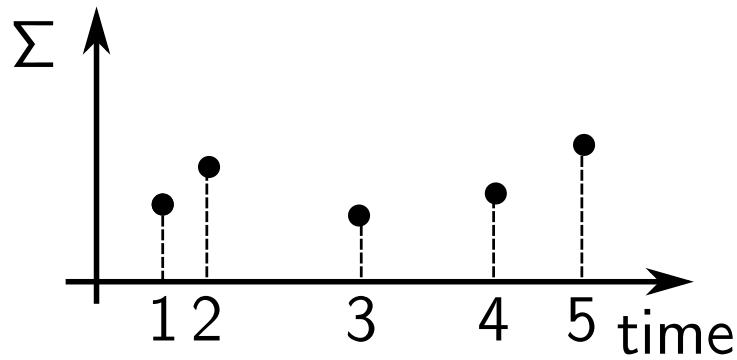
For not-uniformly bounded convergence, one refers to the respective ω-languages and requires

$$\lim K \subseteq \lim(\Sigma^* E)\,.$$

A discrete-event system is a model of a process ... with a particular focus on the ocurrence of events

- finite set Σ of symbols $\sigma \in \Sigma$
- only event ordering is regarded relevant (logic time)
- within finite physical time a finite sequence $s \in \Sigma^*$ is generated
- set $M \subseteq \Sigma^*$ of sequences that can be generated
- write pre M to emphasise that $M = \text{pre}\, M$ (local behaviour)





A closed language pre $M \subseteq \Sigma^*$ is a discrete-event system.

Properties

- safety – bad things never happen

  with $\operatorname{pre} E \subseteq \Sigma^*$, require

  $$\operatorname{pre} M \subseteq \operatorname{pre} E$$

- liveness – good things do happen

  free of deadlocks

  $$(\forall\, s \in \operatorname{pre} M)(\exists\, \sigma \in \Sigma)[\, s\sigma \in \operatorname{pre} M\,]$$

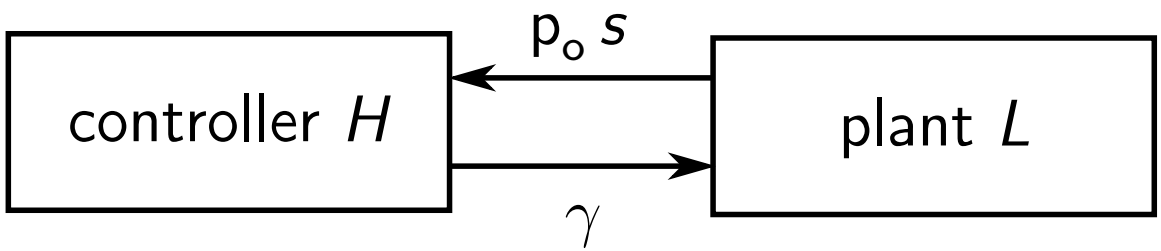
  free of livelocks w.r.t. $L \subseteq \operatorname{pre} M$

  $$(\forall\, s \in \operatorname{pre} M)(\exists\, t \in \Sigma^*)[\, st \in L \cap \operatorname{pre} M\,]$$

For systems with liveness properties:

A language $L \subseteq \Sigma^*$ is a discrete-event system.

W.r.t. the partitioning $\Sigma = \Sigma_c \dot\cup \Sigma_{uc} = \Sigma_o \dot\cup \Sigma_{uo}$ consider a plant $L \subseteq \Sigma^*$ and a controller H in closed-loop configuration:



- at any time, the controller is provided $\mathrm{p}_o\, s \in \Sigma_o^*$ where $s \in \Sigma^*$ is the sequence generated so far;
- in turn, the controller applies a control pattern $\gamma \in \Gamma$ of enabled events, where $\Sigma_{uc} \subseteq \gamma$;
- represent the controller as a discrete-event system

  $$H \subseteq \Sigma^* .$$

  i.e. $\gamma = \{\, \sigma \mid s\sigma \in H \,\}$.

Def. A controller $H \subseteq \Sigma^*$ is *admissible w.r.t. the plant* $L \subseteq \Sigma^*$, if

[H0] $H = \operatorname{pre} H$,

[H1] $(\operatorname{pre} H)\Sigma_{uc} \subseteq \operatorname{pre} H$,

[H2] $\operatorname{pre} H = p_o^{-1}\, p_o \operatorname{pre} H$ (... assuming $\Sigma_c \subseteq \Sigma_o$),

[H3] $(\operatorname{pre} L) \cap (\operatorname{pre} H)$ does not deadlock, and

[H4] $(\operatorname{pre} L) \cap (\operatorname{pre} H) = \operatorname{pre}(L \cap H)$.

Then $K := L \cap H$ represents the cosed-loop behaviour. □

Structural requirement [H4&5]:
liveness properties of the plant shall be retained.

Thm [SCT]: For a plant $L \subseteq \Sigma^*$ and an admissible controller $H \subseteq \Sigma^*$ let $K = L \cap H$. Then

[K0] K is relatively closed w.r.t. L,

[K1] K is controllable w.r.t. L,

[K2] K prefix-normal w.r.t. L, and

[K3] K does not deadlock.

- K is rel. closed w.r.t. L iff
  $K = (\mathrm{pre}\, K) \cap L$
- K is controllable w.r.t. L iff
  $((\mathrm{pre}\, K)\Sigma_{\mathrm{uc}}) \cap (\mathrm{pre}\, L) \subseteq \mathrm{pre}\, K$
- K is prefix normal w.r.t. L iff
  $\mathrm{pre}\, K = (\mathrm{p_o}^{-1}\, \mathrm{p_o}\, \mathrm{pre}\, K) \cap (\mathrm{pre}\, L)$
- K does not deadlock iff
  $\forall\, s \in \mathrm{pre}\, K\, \exists\, \sigma \in \Sigma\colon s\sigma \in \mathrm{pre}\, K$

Vice versa, if K satisfies [K0]-[K3], then there exists an admissible controller H such that $K = L \cap H$. □

Control Problem: given (L, E) with plant $L \subseteq \Sigma^*$ and a specification $E \subseteq \Sigma^*$ construct an admissible controller $H \subseteq \Sigma^*$ such that

$$K := L \cap H \subseteq E\,.$$

Solution: all closed-loop properties are retained under arbitraty union; thus

$$K^{\uparrow} = \sup\{K \subseteq L \cap E \mid K \text{ satisfies [K0]–[K3]}\}$$

itself satisfies [K0]–[K3] and is used to extract a mximally permissive controller.

Note: E can be substituted by a closed language without affecting solutions – it is effectivly a pure safety specification. This becomes a different story when considering ω-languages.

Fault-Tolerant Control

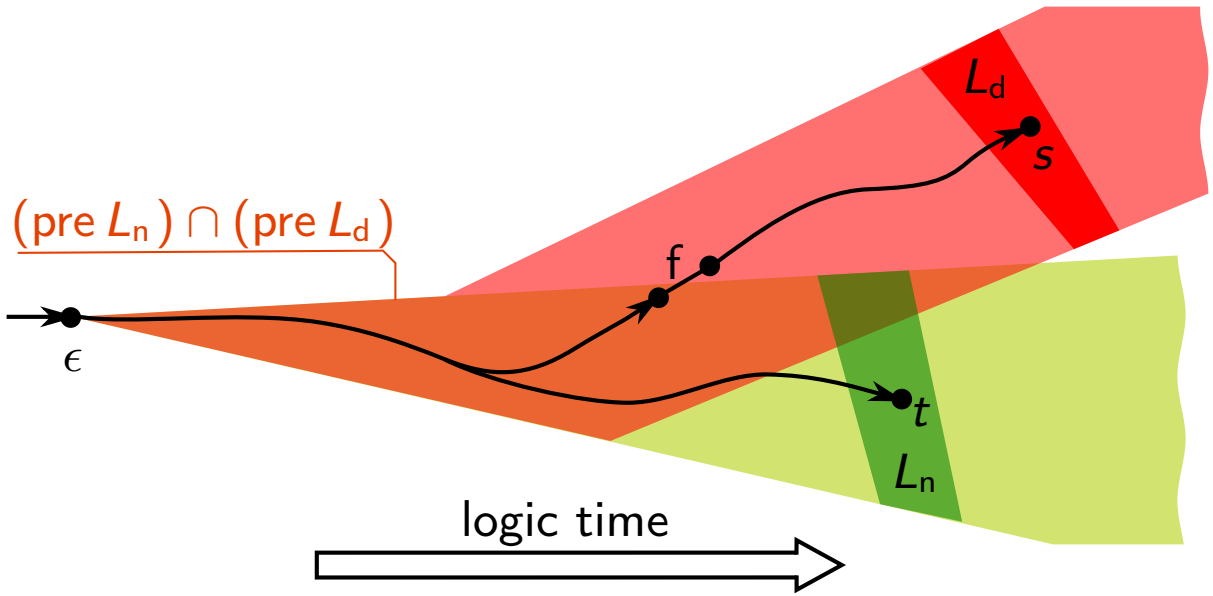
- a fault is a sudden change of behaviour
- passive approach: have a single controller that can handle pre-fault and post-fault behaviour (robust control)
- active approach: detect the fault and switch to another controller (adaptive control)

Core challenge for continuous control systems: switching of plant and controller dynamics and transient behaviour. However, for discrete-event systems:

Sudden change of behaviour and switching in the control scheme are the very nature of discrete-event systems. Hence, fault-tolerant control can be synthesised by the same methods as nominal control [??]

Naive approach to fault-tolerant control:

- nominal plant $L_n \subseteq \Sigma_n^*$
- fault event $f \notin \Sigma_n$, unctrl. and unobs., let $\Sigma_f := \Sigma_n \dot{\cup} \{f\}$
- degraded post-fault behaviour $L_d \subseteq (\mathrm{pre}\, L_n) f \Sigma_f^*$
- *fault-accommodating model* $L_f := L_n \cup L_d$



Algebraic consequence:

Prop. The prerequisite $L_d \subseteq (\text{pre}\, L_n)f\Sigma_f^*$ implies that:

$$L_d \cap \Sigma_n^* = \emptyset\,, \qquad (\text{pre}\, L_d) \cap \Sigma_n^* \subset \text{pre}\, L_n\,,$$
$$L_f \cap \Sigma_n^* = L_n\,, \qquad (\text{pre}\, L_f) \cap \Sigma_n^* = \text{pre}\, L_n\,.$$

From the last line we obtain $(\text{pre}\, L_f) \cap \Sigma_n^* = \text{pre}\, L_n = \text{pre}\,(L_f \cap \Sigma_n^*)$. I.e., the fault-accommodating model and the hypothesis the fault not to occur are non-conflicting. More general we require the fault to never become an inevetible consequence of the past event sequence.

Def. The fault-accommodating model L_f is *well-posed*, if

$$\forall\, s \in \text{pre}\, L_f\, \exists\, \sigma \in \Sigma_n : s\sigma \in \text{pre}\, L_f\,,$$
$$\forall\, s \in \text{pre}\, L_f\, \exists\, t \in \Sigma_n^* : st \in L_f\,.$$

Naive approach to fault-tolerant control (cnt.)

- fault-accommodating specs. $E_\mathrm{f} := E_\mathrm{n} \cup E_\mathrm{d}$, same spirit as L_f
- invoke std. synthesis procedure for $(L_\mathrm{f}, E_\mathrm{f})$
- obtain a controller H_f with sup. closed loop $K_\mathrm{f}^\uparrow = L_\mathrm{f} \cap H_\mathrm{f}$

However: we may encounter

$$\exists\, s \in \operatorname{pre} K_\mathrm{f} \quad \forall\, \sigma \in \Sigma_\mathrm{f} : \quad s\sigma \in \operatorname{pre} K_\mathrm{f} \;\Rightarrow\; \sigma = \mathrm{f}\,,$$
$$\exists\, s \in \operatorname{pre} K_\mathrm{f} \quad \forall\, t \in \Sigma_\mathrm{f}^* : \quad st \in K_\mathrm{f} \;\Rightarrow\; t \notin \Sigma_\mathrm{n}^*\,,$$

This is not desirable – impose additional requirements:

$$\text{[K4]} \quad \forall\, s \in \operatorname{pre} L_\mathrm{f} \quad \exists\, \sigma \in \Sigma_\mathrm{n} \quad : s\sigma \in \operatorname{pre} L_\mathrm{f}\,,$$
$$\text{[K5]} \quad \forall\, s \in \operatorname{pre} L_\mathrm{f} \quad \exists\, t \in \Sigma_\mathrm{n}^* \quad : st \in L_\mathrm{f}\,.$$

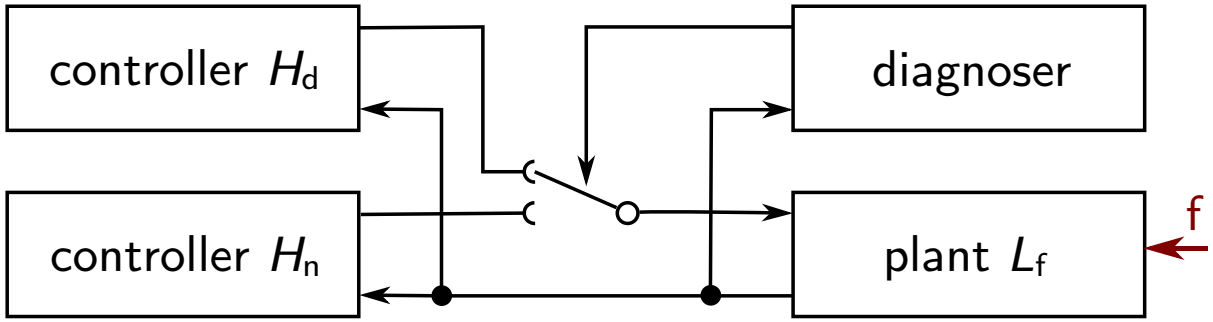
Naive approach to fault-tolerant control (cnt.)

- fault-accommodating specs. $E_\mathrm{f} := E_\mathrm{n} \cup E_\mathrm{d}$, same spirit as L_f
- invoke synthesis procedure for $(L_\mathrm{f}, E_\mathrm{f})$ incl. [K4] and [K5]
- obtain a controller H_f with sup. closed loop $K_\mathrm{f}^\uparrow = L_\mathrm{f} \cap H_\mathrm{f}$

Thm. [N-FTC]: Consider a persistent fault, $L_\mathrm{f} \subseteq \Sigma_\mathrm{n}^* \{\epsilon, \mathbf{f}\} \Sigma_\mathrm{n}^*$. Then there exists a controller H_f with $K_\mathrm{f} = L_\mathrm{f} \cap H_\mathrm{f}$ that is admissible to both L_f and L_n if and only if K_f satiesfies [K0]-[K5].

- diagnosability not required, passive fault-tolerant control
- in general, we have $L_\mathrm{n} \cap H_\mathrm{f} \subseteq K_\mathrm{n}^\uparrow$ — may compute $K_\mathrm{n}^\uparrow$ and test for equality.

Active fault-tolerant control

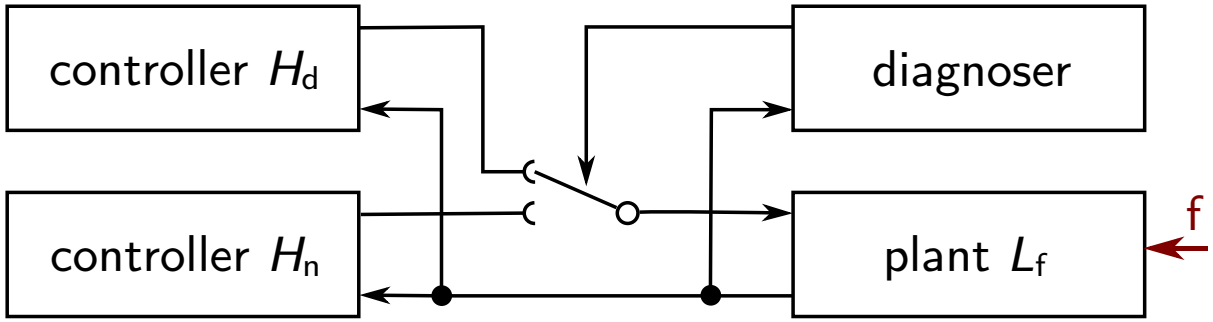


- require the fault to be diagnosable, denote $D \subseteq L_d$ the strings corresponding to f-certain diagnoser states

Diagnosis of DES (Sampath et al 1995)

- *diagnoser*: observer automaton with dedicated state labels
- *f-certain state*: state in which the fault must have occured some time ago
- *diagnosability*: require the plant to after the fault attain an f-certain state after a bounded number of transitions.

Active fault-tolerant control



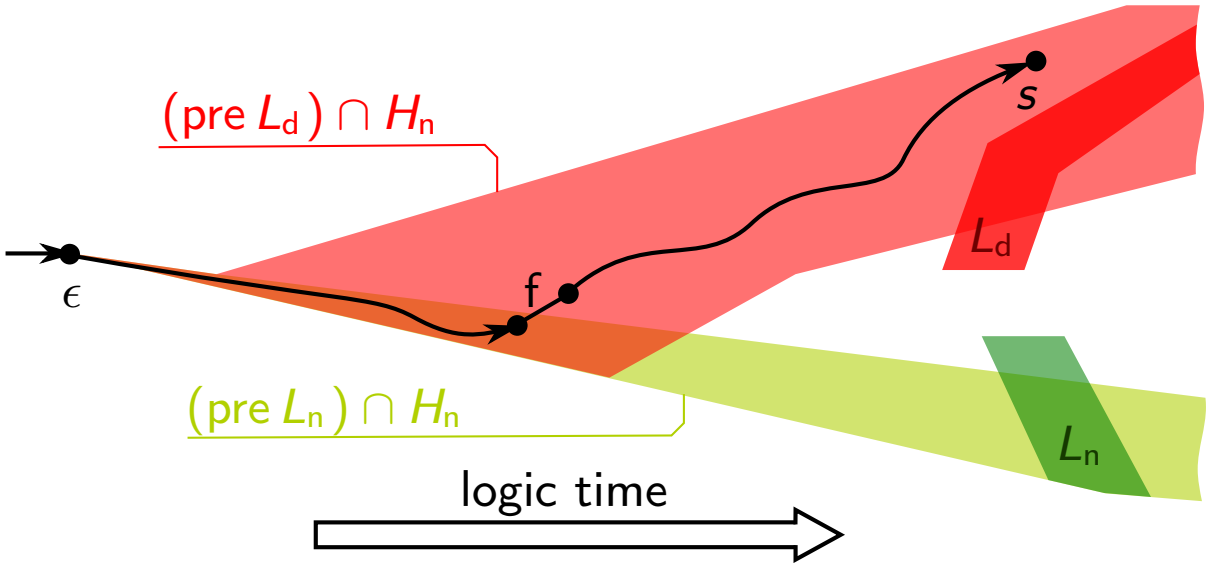
- require the fault to be diagnosable, denote $D \subseteq L_d$ the strings corresponding to f-certain diagnoser states
- require/test that the post-fault-pre-detection behaviour satisfies a safety specification (safe diagnosibility)
- design H_d to take over H_n when the plant first enters D
- note: nominal pre-fault behaviour is guaranteed
- option: synthesise H_d online once the fault has been diagnosed

- associate $H_n \hookleftarrow \mathrm{p}_n^{-1} H_n$ and consider the local-closed loop under nominal control $K_{\mathrm{loc}} := (\mathrm{pre}\, L_f) \cap H_n$
- safe diagnosability condition:

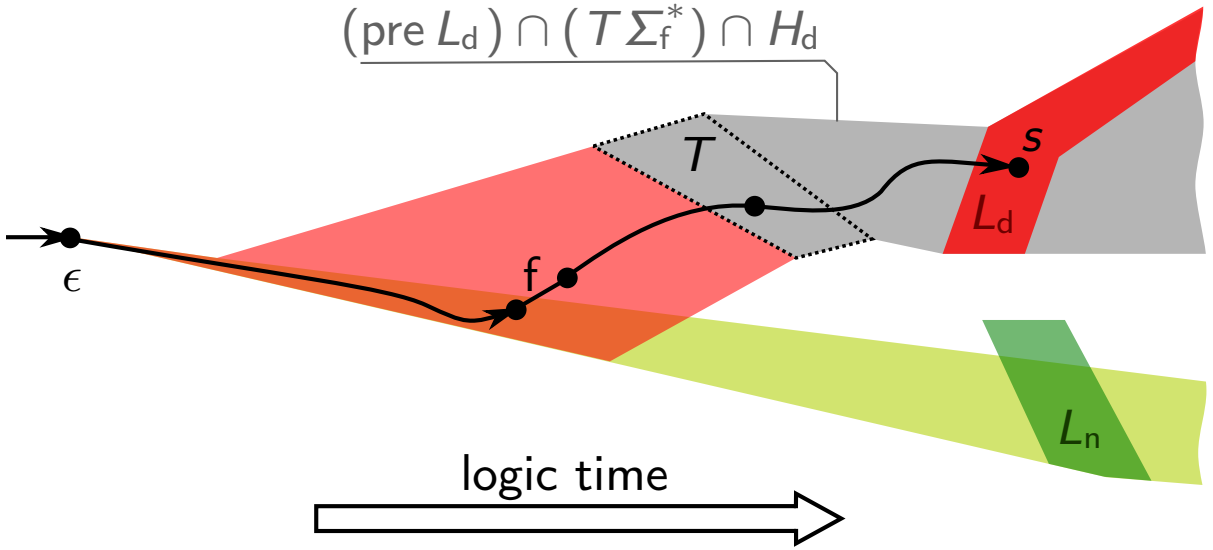
$$D := \{ s \in \Sigma_f^* \mid K_{\mathrm{loc}} \cap (\mathrm{p}_o^{-1} \mathrm{p}_o\, s) \subseteq \Sigma_n^* \mathrm{f} \Sigma_f^* \}\,,$$

$$K_{\mathrm{loc}} \cap \Sigma_n^* \mathrm{f} \Sigma_f^k \subseteq D \text{ for some } k \in \mathbb{N}\,,$$

$$T := \{ s \in K_{\mathrm{loc}} \mid (\mathrm{pre}\, s) \cap D = s \} \subseteq E_{\mathrm{phi}}\,;$$



- local post-fault-detection behaviour $(\mathrm{pre}\, L_\mathrm{d}) \cap (T\Sigma_\mathrm{n}^*)$
- post-fault-detection controller H_d requirements:
  - [A1] admissible w.r.t. $L_\mathrm{d} \cap (T\Sigma_\mathrm{n}^*)$
  - [A2] enforces post fault specs. $L_\mathrm{d} \cap (T\Sigma_\mathrm{n}^*) \cap H_\mathrm{d} \subseteq E_\mathrm{d}$
  - [A3] passive before fault-detection $T \subseteq (\mathrm{pre}\, L_\mathrm{d}) \cap (T\Sigma_\mathrm{n}^*) \cap H_\mathrm{d}$



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- post-fault-detection controller H_d requirements:
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  - [A2] enforces post fault specs. $L_\mathrm{d} \cap (T\Sigma_\mathrm{n}^*) \cap H_\mathrm{d} \subseteq E_\mathrm{d}$
  - [A3] passive before fault-detection $T \subseteq (\mathrm{pre}\, L_\mathrm{d}) \cap (T\Sigma_\mathrm{n}^*) \cap H_\mathrm{d}$
- require that E_d and E_phi relate by

  $$(\mathrm{pre}\, T) \cap (\Sigma_\mathrm{n}^* \mathrm{f} \Sigma_\mathrm{n}^*) \subseteq E_\mathrm{d} \subseteq E_\mathrm{phi}$$

- post-fault-detection controller H_d, synthesis:
  - for [A1] and [A2] use std. procedure on $(L_\mathrm{d} \cap (T\Sigma_\mathrm{n}^*),\, E_\mathrm{d})$
  - then test for [A3]
  - if test fails, no solution exists

Re-interpret active FTC as naive FTC

- formally construct overall controller H_f:

$$H_\mathrm{f} := \{\epsilon\} \cup \{\, s\sigma \in H_\mathrm{n} \,|\, s \notin C \,\} \cup \{\, s\sigma \in H_\mathrm{d} \,|\, s \in C \,\},$$
$$C := (\, D \cap H_\mathrm{n} \cap H_\mathrm{d} \,) \cap \Sigma_\mathrm{f}^* \,.$$

Thm. [A-FTC]: Given a fault-accomodating model L_f and a nominal controller H_n admissible to $L_\mathrm{n} = L_\mathrm{f} \cap \Sigma_\mathrm{n}^*$, assume that local closed loop $K_\mathrm{loc} := (\mathrm{pre}\, L_\mathrm{f}) \cap H_\mathrm{n}$ is safe diagnosable. If a post-fault-detection controller H_d satisfies conditions [A1]–[A3], then the overall controller H_f defined above is admissible to both L_f and L_n with $L_\mathrm{n} \cap H_\mathrm{f} = L_\mathrm{n} \cap H_\mathrm{n}$.

- by Thm. [N-FTC] the conclusion is equivalent to K_f being a closed-loop behaviour achievable by naive FTC.

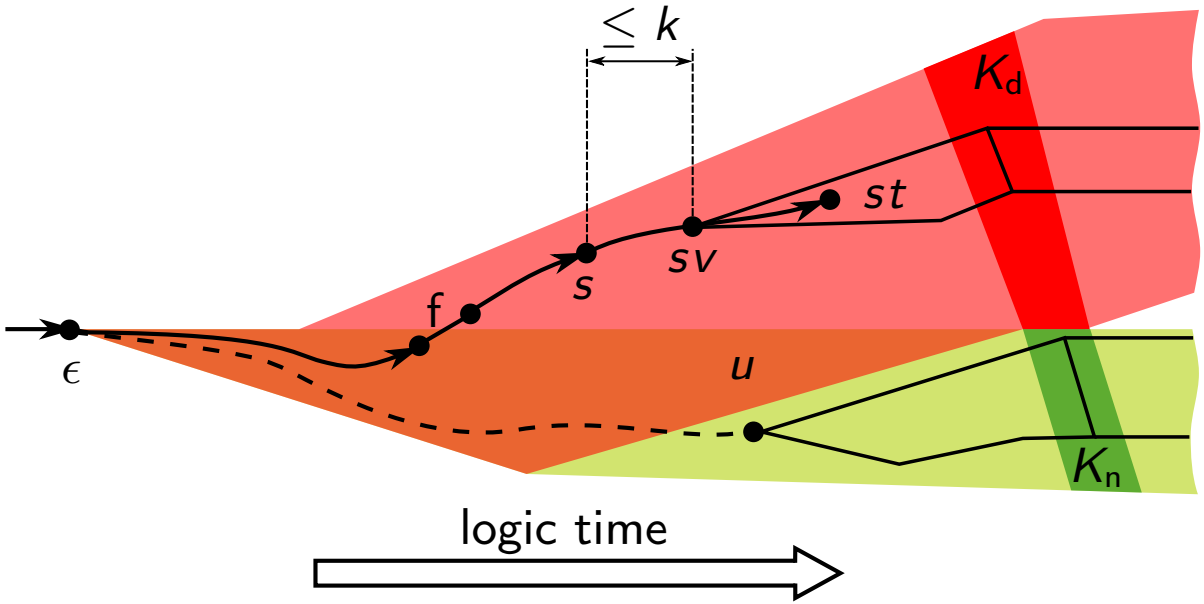
Post-Fault Recovery: safety

Def. [K6]: A closed loop $K_f = K_n \dot{\cup} K_d$ is *weakly recovering* if there exists a uniform bound k such that for all s, t, $|t| \geq k$ with

$$s \in (\operatorname{pre} K_f) \cap (\Sigma_n^* f \Sigma_n^*) \text{ and } st \in \operatorname{pre} K_f$$

there exists $u \in \operatorname{pre} K_n$, $v \in \operatorname{pre} t$, $|v| \leq k$ with

$$K_f/sv \subseteq K_n/u\,.$$



Post-Fault Recovery: safety

Def. [K6]: A closed loop $K_\mathrm{f} = K_\mathrm{n} \dot\cup K_\mathrm{d}$ is *weakly recovering* if there exists a uniform bound k such that for all s, t, $|t| \geq k$ with

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there exists $u \in \operatorname{pre} K_\mathrm{n}$, $v \in \operatorname{pre} t$, $|v| \leq k$ with

$$K_\mathrm{f}/sv \subseteq K_\mathrm{n}/u\,.$$

- synthesis problem: given $L_\mathrm{f} = L_\mathrm{n} \cup L_\mathrm{d}$ and E_f, compute an admissible controller H_f such that the closed loop K_f satisfies [K6].
- the property is not retained under union; synthesis procedure exists for $\Sigma_\mathrm{o} = \Sigma$

Post-Fault Recovery: safety

- Weakly recovering [K6] implies conditional finite convergence:

  [K6'] $K_\mathsf{n}/\Sigma^* \Leftarrow K_\mathsf{f}/(\Sigma_\mathsf{n}^*\mathsf{f})$

- formally generalise to:

  [K6''] $E_\mathsf{f} \Leftarrow K_\mathsf{f}/(\Sigma_\mathsf{n}^*\mathsf{f})$

- synthesis problem: given $L_\mathsf{f} = L_\mathsf{n} \cup L_\mathsf{d}$ and E_f, compute an admissible controller H_f such that the closed loop K_f satisfies [K6']/[K6''].

- neither [K6''] nor [K6'] are retained under union; synthesis procedure exists.

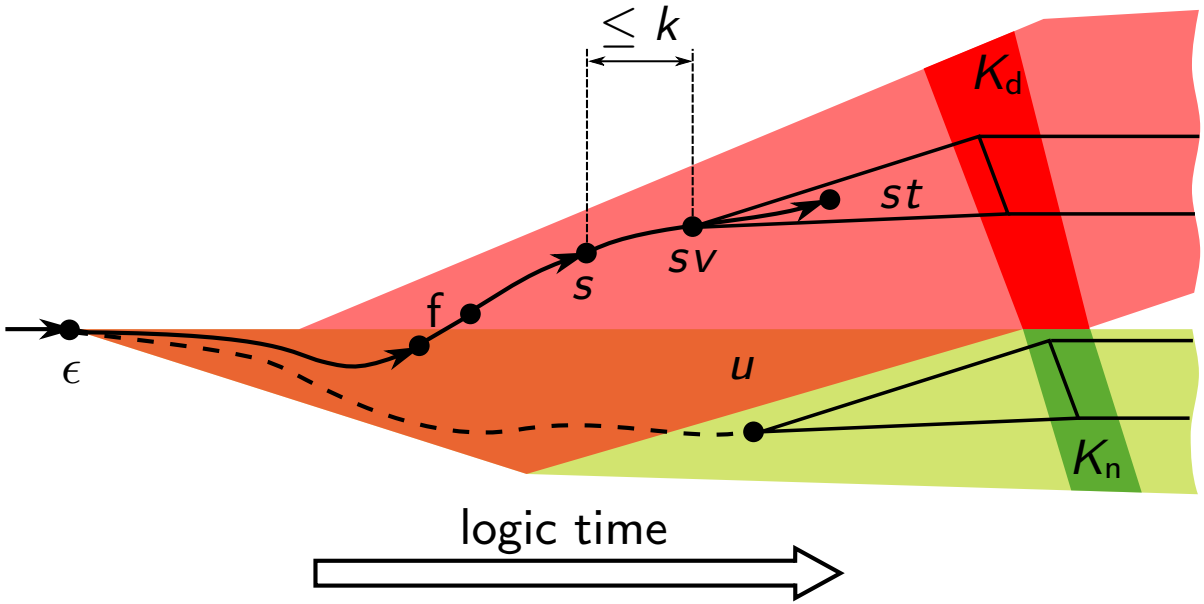
Post-Fault Recovery: liveness

Def. [K7]: A closed loop $K_f = K_n \dot{\cup} K_d$ is *weakly recovering* if there exists a uniform bound k such that for all s, t, $|t| \geq k$ with

$$s \in (\mathrm{pre}\, K_f) \cap (\Sigma_n^* f \Sigma_n^*) \text{ and } st \in \mathrm{pre}\, K_f$$

there exists $u \in \mathrm{pre}\, K_n$, $v \in \mathrm{pre}\, t$, $|v| \leq k$ with

$$K_f/sv = K_n/u\,.$$



Post-Fault Recovery: liveness

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$$s \in (\operatorname{pre} K_f) \cap (\Sigma_n^* f \Sigma_n^*) \text{ and } st \in \operatorname{pre} K_f$$

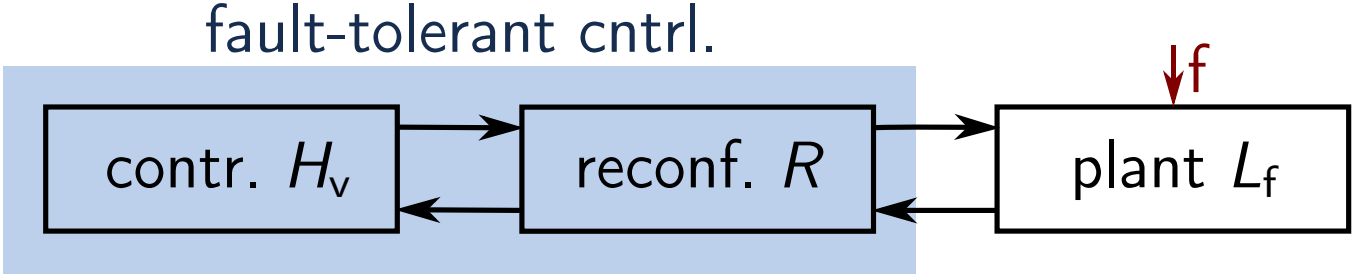
there exists $u \in \operatorname{pre} K_n$, $v \in \operatorname{pre} t$, $|v| \leq k$ with

$$K_f/sv = K_n/u\,.$$

- synthesis problem: given $L_f = L_n \cup L_d$ and E_f, compute an admissible controller H_f such that the closed loop K_f satisfies [K7].
- the property is not retained under union; synthesis procedure exists for $\Sigma_o = \Sigma$

Fault Hiding

Given $L_f = L_n \cup L_d$, $E_f = E_n \cup E_d$, and a solution H_n to (L_n, E_n)



- disconnect nominal controller, i.e., $H_v = h(H_n) \subseteq \Sigma_v^*$ with $\Sigma_v \cap \Sigma_f = \emptyset$, h bijective and applied per event.
- synthesise reconfiguration dynamics $R \subseteq (\Sigma_v \cup \Sigma_o)^*$ to re-connect
- do so by interpreting $H_v \parallel L_f$ as plant and use std. procedures on adapted language inclusion specification, extract R from K
- obtain an overall fault-tolerant controller from H_v and R

- when using a minimal restrictive solution $H_n^{\uparrow}$ resp. $H_v^{\uparrow} \parallel L_f$ for the design, and if the closed loop $K^{\uparrow}$ satisfies [K0]-[K3] and additionally

  [K8] $\quad (\,\forall\, s \in \operatorname{pre} K\,)[\,((\mathrm{p}_v\, s)h(\Sigma_{uc})) \;\cap\; (\operatorname{pre} h(L_n)) \neq \emptyset$
  $\quad\Rightarrow\quad s(\Sigma - h(\Sigma_c))^* h(\Sigma_{uc}) \;\cap\; (\operatorname{pre} K) \neq \emptyset\,]$

  then the corresponding R is admissible to $H_v \parallel L_f$ for any nominal controller H_n that solves (L_n, E_n).

- [K8] is retained under union, synthesis procedures are available.

Note: Nominal controller does not need to be known.

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Summary

Fault-tolerant supervisory control is addressed by the recent literature in various ways, including passive and active approaches, post-fault recovery and fault-hiding.

Conclusions

- switching is addressed by the common modelling framework — any method for fault-tolerant supervisory control should be interpretable within this framework
- additional features of individual approaches amount to additional closed-loop properties — and novel to synthesis problems
- insisting in uniform bounds for diagnosibility and language convergence may be too strict for particular applications — discussion in terms of ω-languages may turn out beneficial

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